The revival-collapse phenomenon in the fluctuations of quadrature field components of the multiphoton Jaynes–Cummings model

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Abstract

In this paper we consider a system consisting of a two-level atom, initially prepared in a coherent superposition of upper and lower levels, interacting with a radiation field prepared in generalized quantum states in the framework of multiphoton Jaynes–Cummings model. For this system, we show that there is a class of states for which the fluctuation factors can exhibit the revival-collapse phenomenon (RCP) similar to that exhibited in the corresponding atomic inversion. This is shown not only for normal fluctuations but also for amplitude-squared fluctuations. Furthermore, apart from this class of states we generally demonstrate that the fluctuation factors associated with three-photon transition can provide the RCP similar to that occurring in the atomic inversion of the one-photon transition. These are novel results and their consequence is that the RCP occurring in the atomic inversion can be measured via a homodyne detector. Furthermore, we discuss the influence of the atomic relative phases on such phenomenon.

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1. Introduction

Interaction between the radiation field and matter is an important topic in modern physics. One of the most important systems, describing well the field–matter interaction, is the Jaynes–Cummings model (JCM). The JCM consists of a single two-state system interacting with a single quantized radiation field mode [1]. Furthermore, the JCM has become experimentally realizable with the Rydberg atoms in high-Q microwave cavities (e.g., see [2]). Moreover, the JCM is exactly solvable under the rotating wave approximation and many interesting phenomena have been observed. The most important phenomenon is the behaviour of the population inversion where instead of displaying steady Rabi oscillations in the case of a

classical field coupled to the atom [3], there is an initial collapse of these oscillations followed by regular revivals that slowly become broader and eventually overlap [4]. In fact, the revival-collapse phenomenon (RCP) of the atomic inversion is a pure quantum mechanical effect having its origin in the granular structure of the photon-number distribution of the initial field [5]. The features of the RCP for the JCM have been analysed in details in [4]. Moreover, it has been shown that the envelope of each revival is a readout of the photon distribution, in particular, for the states whose photon-number distributions are slowly varying [6]. It is worth mentioning that observation of the RCP has been performed using the one-atom mazer [2], which is more sophisticated than the dynamics of the JCM.

On the other hand, quadrature fluctuations of the radiation field are important quantities in quantum optics, which can be measured by a homodyne detection in which the signal is superimposed on a strong coherent beam of the local oscillator. The question we would like to address here is: can the quadrature fluctuations of the multiphoton JCM include information on the RCP of the atomic inversion? If it is so then the RCP can be detected via a homodyne detector. In other words, the quadrature fluctuations as well as atomic inversion of the JCM can be measured by means of one device. In this case the scheme will be simple, involving one beam splitter and a reference field in a coherent state. In the present paper, we show that such behaviour can occur. Specifically, we show that the radiation-field fluctuation (i.e. squeezing) factors of the cubic JCM can carry information on the atomic inversion of the standard JCM (i.e. a JCM which involves one photon for making atomic transition) for the same initial states. Moreover, we show that there is a class of states whose fluctuation factors can include explicit information on the RCP. Furthermore, we demonstrate that such phenomenon can occur in the higher-order fluctuation, e.g., also amplitude-squared fluctuations. In fact, these are novel results and may be useful for experimentalists. We have to stress that in this paper we are not looking for squeezing of the JCM, which has been intensively studied by several authors (e.g., see [7, 8]). Nevertheless, we look at the occurrence of the RCP in the fluctuation factors. This will be investigated in the following order. In section 2, we give the basic calculations related to the system under consideration. In sections 3 and 4, we discuss the occurrence of the RCP in the normal fluctuations and amplitude-squared fluctuations, respectively. The results are summarized in section 5.

2. General considerations

In this section, we give both the explicit form for the Hamiltonian of the system under consideration and the basic calculations related to such system. The system considered in this paper is the multiphoton resonance interaction of a single-mode field with a two-level atom, which is described by the mth-photon JCM. The effective Hamiltonian controlling the system in the rotating wave approximation (RWA) is [9]

$$\frac{\hat{H}}{\hbar} = \omega_0 \hat{a}^\dagger \hat{a} + \omega_a \hat{\sigma}_z + \lambda(\hat{a}^m \hat{\sigma}_+ + \hat{a}^{\dagger m} \hat{\sigma}_-), \qquad (1)$$

where $\hat{\sigma}_\pm$ and $\hat{\sigma}_z$ are the Pauli spin operators, ω_0 and ω_a are the frequencies of cavity mode and the atomic transitions, respectively, λ is the atom-field coupling constant and m is the number of photons involved in the atomic transition. Defining two new operators as

$$\hat{C}_1 = \omega_0 \hat{a}^\dagger \hat{a} + \omega_a \hat{\sigma}_z, \qquad \hat{C}_2 = \lambda(\hat{a}^m \hat{\sigma}_+ + \hat{a}^{\dagger m} \hat{\sigma}_-). \qquad (2)$$

In the exact resonance case (i.e. $\omega_a = m\omega_0$) it is easy to prove that $\hat{C}_1$ and $\hat{C}_2$ are constants of motion and also they commute with each other. This fact means that the evolution of the mean-photon number and the atomic inversion of the system include typical information on

each other. In the interaction picture, the unitary evolution operator takes the form

$$\hat{U}_I(T, 0) = \exp\left(-i\frac{T}{\lambda}\hat{C}_2\right)$$

$$= \cos(T\hat{D}) - i\frac{\sin(T\hat{D})}{\lambda\hat{D}}\hat{C}_2, \tag{3}$$

where

$$T = \lambda t, \qquad \hat{D}^2 = \hat{a}^{\dagger m}\hat{a}^m\hat{\sigma}_-\hat{\sigma}_+ + \hat{a}^m\hat{a}^{\dagger m}\hat{\sigma}_+\hat{\sigma}_-. \tag{4}$$

It is worth recalling that $\hat{\sigma}_\pm^2 = 0$.

On the other hand, to keep the analysis quite general, we consider the field prepared initially in a general pure quantum state described by

$$|\psi(0)\rangle = \sum_{n=0}^{\infty} C_n|kn\rangle, \tag{5}$$

where C_n represent the probability amplitudes for the state under consideration such that $\sum_{n=0}^{\infty} |C_n|^2 = 1$, and k is a parameter whose value will be specified in the text. Throughout the paper, we consider the probability amplitudes C_n to be real. It is worth mentioning that when C_n represent the probability amplitudes of the well-known Gluaber coherent state and $k \neq 1$, then (5) gives the k-photon coherent states [10, 11]. These states are obtained from kth harmonic generation using Brandt–Greenberg operators [12]. It has been shown that such a class of states can exhibit amplitude kth-power squeezing [11] when they interact with the nonlinear nonabsorbing medium modelled as an anharmonic oscillator. We proceed by considering that the atom is initially in the coherent superposition of the excited and ground states as

$$|\theta, \phi\rangle = \cos\theta|+\rangle + \exp(-i\phi)\sin\theta|-\rangle, \tag{6}$$

where $|+\rangle$ and $|-\rangle$ denote excited and ground atomic states, respectively; θ and ϕ are the relative phases between these two atomic states. Actually, preparing the atom in the coherent superposition states is important because of its applications to noise quenching by correlated spontaneous emission [13], quantum beats [14] and noise-free amplification [15].

Now the initial state of the field–atom system can be expressed as

$$|\Psi(0)\rangle = |\psi(0)\rangle \bigotimes |\theta, \phi\rangle. \tag{7}$$

Therefore, the dynamical wavefunction of the total system in the interaction picture is given by

$$|\Psi(T)\rangle = \hat{U}_I(T, 0)|\Psi(0)\rangle$$

$$= \sum_{n=0}^{\infty}[G_1(n, T)|+, n\rangle + G_2(n, T)|-, n+m\rangle], \tag{8}$$

where

$$G_1(n, T) = C_n\cos\theta\cos(T\sqrt{h(n, m)}) - i\exp(-i\phi)C_{n+m}\sin\theta\sin(T\sqrt{h(n, m)}),$$
$$G_2(n, T) = \exp(-i\phi)C_{n+m}\sin\theta\cos(T\sqrt{h(n, m)}) - iC_n\cos\theta\sin(T\sqrt{h(n, m)}), \tag{9}$$

while $h(n, m) = \frac{(n+m)!}{n!}$ and in the course of the calculation we have considered $k = 1$ (cf (5)). For future purposes, we derive different moments for the $\hat{a}^\dagger$ and $\hat{a}$ associated with the

state (8) as

$$
\langle \hat{a}^{\dagger s_2}(T)\hat{a}^{s_1}(T)\rangle = \sum_{n=0}^{\infty}\left[G_1^*(n+s_2,T)G_1(n+s_1,T)\frac{\sqrt{(n+s_1)!(n+s_2)!}}{n!}\right.
$$
$$
\left. + G_2^*(n+s_2,T)G_2(n+s_1,T)\frac{\sqrt{(n+m+s_1)!(n+m+s_2)!}}{(n+m)!}\right],
\tag{10}
$$

where s_1 and s_2 are positive integers. Also the atomic inversion for the dynamical state (8) is

$$
\langle \sigma_z(T)\rangle = \sum_{n=0}^{\infty}\{[P(n)\cos^2\theta - P(n+m)\sin^2\theta]\cos(2T\sqrt{h(n,m)})
$$
$$
- \sqrt{P(n)P(n+m)}\sin\phi\sin(2\theta)\sin(2T\sqrt{h(n,m)})\},
\tag{11}
$$

where $P(n) = C_n^2$.

We conclude this section by mentioning that, along the lines of [16], the use of the Hamiltonian (1) is called an effective Hamiltonian approach (EHA). Nevertheless, the full microscopic Hamiltonian approach (FMHA) associated with the system can be obtained by considering the Hamiltonian, which describes the interaction between $(m+1)$th-level atom in a cascade configuration with the single-mode radiation field in the RWA [17]. Under certain conditions, the intermediate levels can be cancelled out adiabatically and the system reduced to that of the two-level atom. In this case, the probability amplitudes of the dynamical wavefunction of the system include nontrivial overall phase depending on the intensity of the field. This makes the results associated with the FMHA completely different from those with the EHA, in particular, quantities that depend on the off-diagonal elements of the reduced density matrix such as the quadrature fluctuations. Alternatively, the Hamiltonian (1) can be modified to provide similar information—under certain conditions—as that of the FMHA [16]. This can be achieved by including the dynamic Stark shift in (1), i.e. including such a term $-\hat{a}^{\dagger}\hat{a}(\beta_1\hat{\sigma}_+\hat{\sigma}_- + \beta_2\hat{\sigma}_-\hat{\sigma}_+)$ in (1) where β_1, β_2 are dynamic Stark shift parameters. This technique is called the modified effective Hamiltonian approach (MEHA) and for the sake of comparison we give some details about it. For instance, the dynamical state for the system associated with the MEHA in the interaction picture (considering the initial condition (7)) is

$$
|\tilde{\Psi}(T)\rangle = \sum_{n=0}^{\infty}[\tilde{G}_1(n,T)|+,n\rangle + \tilde{G}_2(n,T)|-,n+m\rangle],
\tag{12}
$$

where

$$
\tilde{G}_1(n,T) = \exp(\mathrm{i}t V_n)\left\{ C_n\cos\theta\cos(t\Omega_n)\right.
$$
$$
\left. + \frac{\mathrm{i}}{\Omega_n}\left[(n\beta_1 - V_n)C_n\cos\theta - \lambda\sqrt{\frac{(n+m)!}{n!}}\exp(-\mathrm{i}\phi)C_{n+m}\sin\theta\right]\sin(t\Omega_n)\right\},
$$
$$
\tilde{G}_2(n,T) = \exp(\mathrm{i}t V_n)\left\{ \exp(-\mathrm{i}\phi)C_{n+m}\sin\theta\cos(t\Omega_n)\right.
\tag{13}
$$
$$
- \frac{\mathrm{i}}{\Omega_n}\left[(V_n - (n+m)\beta_2)\exp(-\mathrm{i}\phi)C_{n+m}\sin\theta\right.
$$
$$
\left.\left. + \lambda\sqrt{\frac{(n+m)!}{n!}}C_n\cos\theta\right]\sin(t\Omega_n)\right\}
$$

and

$$V_n = \frac{1}{2}[n\beta_1 + (n+m)\beta_2], \qquad \Omega_n = \frac{1}{2}\left[(n\beta_1 - (n+m)\beta_2)^2 + 4\lambda\frac{(n+m)!}{n!}\right]^{\frac{1}{2}}. \qquad (14)$$

When $m = 2$ expressions (13) and (14) reduce to (40)–(43) in [16]. However, there is a misprint in (41) of [16] where the term $(n\beta_1 - V_n)$ has to be $(V_n - (n+2)\beta_2)$. Comparison between (9) and (13) shows that involving the dynamic Stark shift in the effective Hamiltonian makes the probability amplitudes including nontrivial overall phase, which depends on the intensity of the field, as we mentioned above in relation to the FMHA.

Throughout the paper, we focus the attention on the EHA. To be more specific, we use expressions (10) and (11) to make a comparative study between the behaviour of the fluctuation factors and atomic inversion. Also, we give only some comments on the MEHA aimed at showing the differences between the EHA and MEHA. So the discussion is generally given for the EHA, except specifying that it is related to the MEHA.

3. The revival-collapse phenomenon in normal fluctuations

In this section, we show that information stored in $\langle\hat{\sigma}_z(T)\rangle$ can be obtained from fluctuation factors of the second-order (normal) fluctuation. To do so we define two quadrature operators as $\hat{X} = \frac{1}{2}[\hat{a}+\hat{a}^\dagger]$, $\hat{Y} = \frac{1}{2i}[\hat{a}-\hat{a}^\dagger]$. These quadratures satisfy the commutation rule $[\hat{X}, \hat{Y}] = \frac{i}{2}$ and thus the uncertainty relation is $\langle(\triangle\hat{X}(T))^2\rangle\langle(\triangle\hat{Y}(T))^2\rangle \geqslant \frac{1}{16}$. Therefore, the fluctuation factors associated with the quadratures $\hat{X}$ and $\hat{Y}$, respectively, read

$$F_1(T) = 2\langle(\triangle\hat{X}(T))^2\rangle - \frac{1}{2} = \langle\hat{a}^\dagger(T)\hat{a}(T)\rangle + \text{Re}\langle\hat{a}^2(T)\rangle - 2(\text{Re}\langle\hat{a}(T)\rangle)^2,$$
$$S_1(T) = 2\langle(\triangle\hat{Y}(T))^2\rangle - \frac{1}{2} = \langle\hat{a}^\dagger(T)\hat{a}(T)\rangle - \text{Re}\langle\hat{a}^2(T)\rangle - 2(\text{Im}\langle\hat{a}(T)\rangle)^2. \qquad (15)$$

The system is able to yield normal squeezing when $F_1(T) < 0$ or $S_1(T) < 0$: however, this is not the aim of this paper. Based on (15) we illustrate that there are two approaches, namely, natural phenomenon and numerical simulation, which can provide the RCP in $F_1(T)$ and/or in $S_1(T)$. In the first approach, we show that there is particular class of states that can naturally exhibit the RCP in the fluctuation factors. Nevertheless, in the second approach we demonstrate that $S_1(T)$, for particular values of m, can exhibit similar behaviour as that of $\langle\hat{\sigma}_z(T)\rangle$ of the standard JCM. In fact these two approaches are related to two different situations in which different terms dominate the variance of the field amplitude. To be more specific, for the natural phenomenon the origin of the RCP in the normal fluctuation is $\langle\hat{a}^\dagger(T)\hat{a}(T)\rangle$; however, that in the numerical simulation approach is $\text{Re}\langle\hat{a}^2(T)\rangle$, as we will show below. Furthermore, we investigate the influence of the atomic relative phases on the occurrence of the RCP in the fluctuation factors. Also we make some comments on the differences between the EHA and MEHA related to the phenomenon under consideration. These points will be investigated in the following two subsections.

3.1. Natural phenomenon

This approach is based on the fact that $\hat{C}_1$ is a constant of motion and then the evolution of the $\langle\hat{a}^\dagger(T)\hat{a}(T)\rangle$ and $\langle\hat{\sigma}_z(T)\rangle$ for the same value of m yields similar behaviour. So that if there are states for which

$$\langle\hat{a}(T)\rangle = 0, \qquad \langle\hat{a}^2(T)\rangle = 0, \qquad (16)$$

simultaneously then the two fluctuation factors in (15) reduce to $\langle\hat{a}^\dagger(T)\hat{a}(T)\rangle$. In other words, $F_1(T)$ and/or $S_1(T)$ provide an information on the atomic inversion. Now we are looking

for such types of state. For convenience we restrict the analysis to $m = 1$ and $\theta = 0$. The associated quantities with this case can be obtained from (10) as

$$
\langle \hat{a}(T) \rangle = \sum_{n=0}^{\infty} C_n C_{n+1} \sqrt{n+1} \left\{ \cos[T\sqrt{h(n,1)}] \cos[T\sqrt{h(n+1,1)}] \right.
$$
$$
\left. + \sqrt{\frac{n+2}{n+1}} \sin[T\sqrt{h(n,1)}] \sin[T\sqrt{h(n+1,1)}] \right\},
$$

$$
\langle \hat{a}^2(T) \rangle = \sum_{n=0}^{\infty} C_n C_{n+2} \sqrt{(n+1)(n+2)} \left\{ \cos[T\sqrt{h(n,1)}] \cos[T\sqrt{h(n+2,1)}] \right.
$$
$$
\left. + \sqrt{\frac{n+3}{n+1}} \sin[T\sqrt{h(n,1)}] \sin[T\sqrt{h(n+2,1)}] \right\}.
$$

$$(17)$$

It is obvious that conditions (16) are satisfied simultaneously only when

$$
C_n C_{n+1} = 0, \qquad C_n C_{n+2} = 0 \tag{18}
$$

and these equalities can be satisfied for three-photon states, four-photon states and so on. The k-photon coherent states (cf (5)) can play this role, e.g. when $k = 3, 4, \ldots$, etc. It is worth mentioning that the properties of the three-photon states have been investigated in [18]. Further, examples of the four-photon states are the orthogonal-even, (-odd) coherent states [19] and phased generalized binomial states [20]. Here we shed light on the behaviour of $F_1(T)$ of the JCM against the orthogonal-even coherent states. Their forms can be obtained from (5) by setting $k = 1$ and replacing the probability amplitudes C_n by

$$
C_{2n} = B \frac{\alpha^{2n}}{\sqrt{(2n)!}} [1 + (-1)^n], \tag{19}
$$

where B is the normalization constant having the form

$$
B = [2 \cosh|\alpha|^2 + 2 \cos|\alpha|^2]^{-\frac{1}{2}}. \tag{20}
$$

Such types of state have been investigated in [19] showing that they cannot exhibit second-order squeezing, whereas near-optimal simultaneous-quadrature fourth-order squeezing can be obtained. Also they can be generated using conditional-measurement technique [21, 22]. Figure 1 has been plotted for $F_1(T)$ of the EHA with the field initially in orthogonal-even coherent states for given values of the parameters. From this figure it is clear that the RCP is established. In fact, the revivals are four times compared to those of the corresponding initial coherent light since orthogonal-even coherent states are a superposition of four-component coherent states. This leads to $T_R^{(f)} = T_R^{(c)}/4$, where $T_R^{(c)}$ and $T_R^{(f)}$ are the revival times associated with the initial coherent states and orthogonal-even coherent states, respectively. This fact can be deduced as follows. For the initial coherent light the revivals occur by estimating the time that neighbour terms in the sums are in phase (for $\bar{n} = \sqrt{\langle \hat{n}(0) \rangle}$, where $\hat{n}(0) = \hat{a}^\dagger(0)\hat{a}(0)$):

$$
2T_R^{(c)} [\sqrt{\langle \hat{n}(0) \rangle + 1} - \sqrt{\langle \hat{n}(0) \rangle}] \simeq 2\pi. \tag{21}
$$

Nevertheless, orthogonal-even coherent states are four-photon state and thus the difference in phase of two (non-zero) neighbour terms will be

$$
2T_R^{(f)} [\sqrt{\langle \hat{n}(0) \rangle + 4} - \sqrt{\langle \hat{n}(0) \rangle}] \simeq 2\pi. \tag{22}
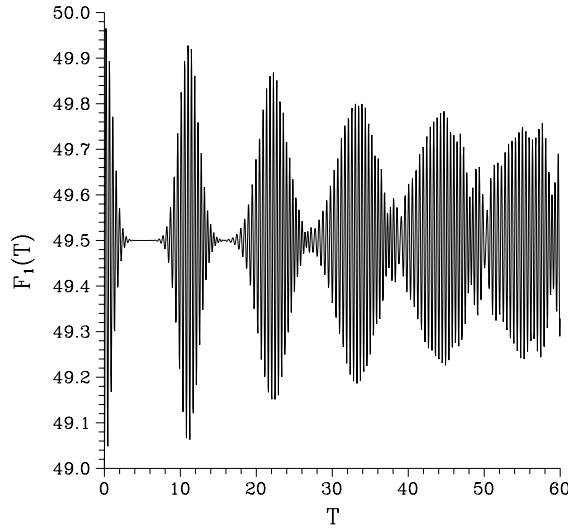$$

Figure 1. The fluctuation factor $F_1(T)$ of the standard JCM against the scaled time T when the optical cavity field initially prepared in orthogonal-even coherent states and the atom is in the excited atomic state for $|\alpha| = 7$.

Expressions (21) and (22) lead to

$$T_R^{(c)} \simeq 2\pi \sqrt{\langle \hat{n}(0) \rangle}, \qquad T_R^{(f)} \simeq \frac{\pi}{2} \sqrt{\langle \hat{n}(0) \rangle}. \tag{23}$$

This means that $T_R^{(f)} = T_R^{(c)}/4$.

The influence of the atomic relative phases on the behaviour of $F_1(T)$ for the present approach can be investigated as follows. As is well known—for the standard JCM and for certain choice of the atomic phases (i.e. for θ and ϕ)—'coherent trapping' occurs [23]. Actually, similar conclusion can be given here, i.e. the interaction has a little effect on $F_1(T)$. For example, for orthogonal-even coherent states this can occur when $\theta = \pi/4$, $\phi = 0$ and $m = 4$. The origin in taking $m = 4$ is quite obvious from (11), where atomic trapping occurs when $\langle \hat{\sigma}_z(T) \rangle \simeq 0$ (or in the language of the present approach when $F_1(T) \simeq \langle \hat{n}(0) \rangle$), i.e.

$$P(n) - P(m+n) \simeq 0. \tag{24}$$

Expression (24) leads to $P(n) \simeq P(n+m)$, i.e. the two successive non-zero values of the photon-number distribution should be comparable. This occurs when m is equal to the parity of the initial state of the optical cavity field. More illustratively, atomic trapping for the mth JCM with optical cavity field prepared initially in, e.g., one-, two-, three- and four-photon states occurs only when $m = 1, 2, 3$ and 4, respectively.

We conclude this section by the following remark. For the natural phenomenon approach, the EHA and MEHA provide almost similar behaviour in relation to the RCP in, e.g., $F_1(T)$. In this case the nonvanishing term (i.e. the mean-photon number) depends only on the diagonal elements of the density matrix and then the intensity-dependent phases in the MEHA are cancelled out. We should point out that the RCP can occur in the fluctuation factors for strong-intensity regime $\langle \hat{n}(0) \rangle \gg 1$, which is the same condition for the EHA and MEHA to provide similar behaviour [16].

3.2. Numerical simulation

In this section, we discuss the possibility of obtaining the RCP from the second-order fluctuation factors of the mth ($m > 2$) JCM similar to that of $\langle \hat{\sigma}_z(T) \rangle$ of the standard JCM, which will be denoted by $\langle \hat{\sigma}_z(T) \rangle_{m=1}$. We assume that the initial states are not those for which the natural phenomenon can occur. Careful examination of (15) shows that the RCP can occur in $F_1(T)$ (or $S_1(T)$) provided that the values of $\text{Re}\langle \hat{a}(T) \rangle$ (or $\text{Im}\langle \hat{a}(T) \rangle$) are approximately zero in the course of the interaction since these quantities are squared and then they spoil the RCP (if it exists). On the other hand, for $m > 2$, $\langle \hat{a}^\dagger(T)\hat{a}(T) \rangle$ exhibits chaotic behaviour (see figure 3(a)). Therefore, under these circumstances, if $S_1(T)$, say, can exhibit the RCP then the origin is in $\text{Re}\langle \hat{a}^2(T) \rangle$. For this reason, we compare the form of $\text{Re}\langle \hat{a}^2(T) \rangle$ with that of $\langle \hat{\sigma}_z(T) \rangle_{m=1}$ for optical cavity field initially in coherent states (with real probability amplitudes) and the atom in the atomic excited state. The aim of such comparison is two-fold: (i) To find the exact values of the number of photons involved in the atomic transition, i.e. m, for which such phenomenon can occur. (ii) To explore the form of the modified fluctuation factor, which can include typical information on the behaviour of the $\langle \hat{\sigma}_z(T) \rangle_{m=1}$. Now from (10) we arrive at

$$\langle \hat{a}^2(T) \rangle = \langle \hat{n}(0) \rangle \sum_{n=0}^{\infty} P(n) \left[\left\{ \sqrt{\frac{(n+m+2)(n+m+1)}{(n+2)(n+1)}} \right\} \right.$$
$$\times \sin(T\sqrt{h(n,m)}) \sin(T\sqrt{h(n+2,m)})$$
$$\left. + \cos(T\sqrt{h(n,m)}) \cos(T\sqrt{h(n+2,m)}) \right], \tag{25}$$

where $P(n)$ is the photon-number distribution for the coherent light and $\langle \hat{n}(0) \rangle = |\alpha|^2$. We treat the problem in a strong-intensity regime when m is finite. In this case the terms contributing effectively to the summation in (25) are those for which $\alpha^2 \simeq n$. Therefore, the square root included in the curly brackets in (25) tends to unity and thus reads

$$\langle \hat{a}^2(T) \rangle = \langle \hat{n}(0) \rangle \sum_{n=0}^{\infty} P(n) \cos[T(\sqrt{h(n+2,m)} - \sqrt{h(n,m)})]. \tag{26}$$

On the other hand, the corresponding atomic inversion of the standard JCM is

$$\langle \hat{\sigma}_z(T) \rangle = \sum_{n=0}^{\infty} P(n) \cos(2T\sqrt{n+1}). \tag{27}$$

Apart from the constant quantity $\langle \hat{n}(0) \rangle$ in (26), expressions (26) and (27) yield similar behaviour provided that the arguments of the $\cos(\cdot)$ are comparable. Therefore, we adopt the following proportionality factor:

$$f(n) = \frac{\sqrt{h(n+2,m)} - \sqrt{h(n,m)}}{2\sqrt{n+1}}. \tag{28}$$

After straightforward calculation (28) takes the form

$$f(n) = \frac{n^{\frac{m-3}{2}} \left[2m + \frac{m}{n}(m+3) \right] \sqrt{\prod_{j=0}^{m-1} \left(1 + \frac{m-j}{n}\right)}}{2\sqrt{1 + \frac{1}{n}} \left[\sqrt{\left(1 + \frac{m+1}{n}\right)\left(1 + \frac{m+2}{n}\right)} + \sqrt{\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)} \right]}. \tag{29}$$

In the strong-intensity regime, expression (29) reduces to

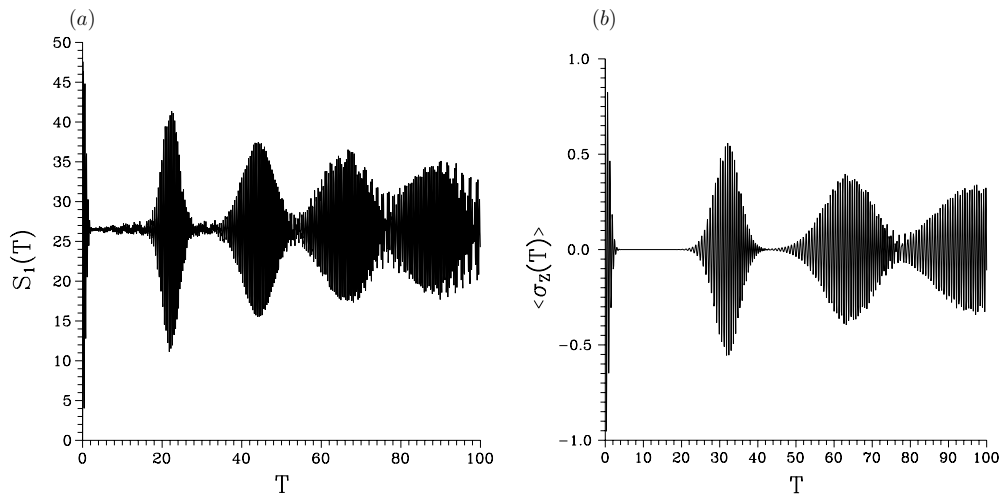$$f(n) \simeq \frac{m}{2} n^{\frac{m-3}{2}}. \tag{30}$$

Figure 2. The fluctuation factor $S_1(T)$ of the JCM for $m = 3$ (a) and the atomic inversion for $m = 1$ (b) against the scaled time T when the field prepared initially in the coherent state with $|\alpha| = 5$ and the atom in the atomic excited state $\theta = 0$.
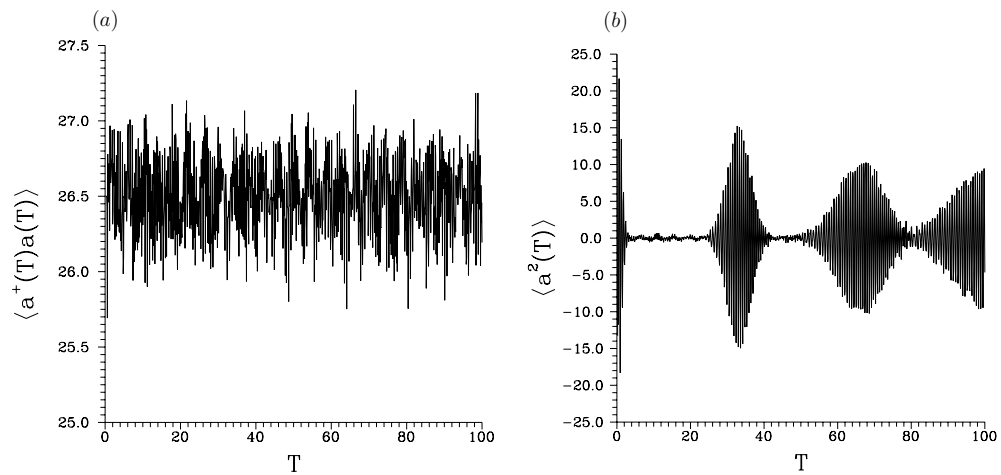

Figure 3. The mean-photon number (a) and the moment $\langle \hat{a}^2(T) \rangle$ (b) against the scaled time T for the same values of the parameters as those in figure 2 but with $m = 3$.

It is evident from (30) that the allowed value of m for which the RCP can occur in $S_1(T)$ is only $m = 3$ and thus $f(n) \simeq 3/2$. The validity of the above facts has been checked numerically in figures 2 and 3.

Figures 2(a) and (b) have plotted for $S_1(T)$ and $\langle \hat{\sigma}_z(T) \rangle$, respectively, for given values of the interaction parameters. According to the above discussion, the RCP can occur only in $S_1(T)$ (since in this case $\text{Re}\langle \hat{a}(T) \rangle \neq 0$ and $\text{Im}\langle \hat{a}(T) \rangle = 0$). Comparison between figures 2(a) and (b) shows that they roughly exhibit similar behaviour in a sense that they revive, collapse, remain quiescent, revive, collapse and so on. For large interaction, time overlapping between successive revivals occurs. Nevertheless, they include different scales, which we treat shortly. Figures 3(a) and (b) shed light on the evolution of the $\langle \hat{a}^{\dagger}(T)\hat{a}(T) \rangle$
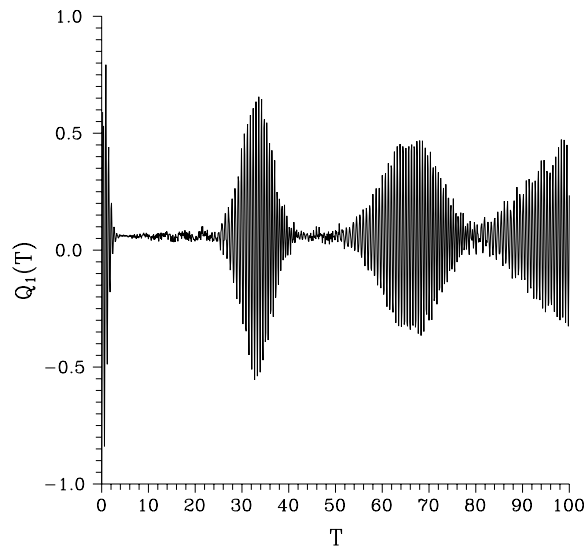
Figure 4. The rescaled fluctuation factor $Q_1(T)$ given by (28) for the same values of the parameters as in figure 2(a).

and $\mathrm{Re}\langle\hat{a}^2(T)\rangle (=\langle\hat{a}^2(T)\rangle)$, respectively, i.e. the non-vanishing components in $S_1(T)$. In these figures the values of the parameters are the same as those in figure 2(a). It is obvious that $\mathrm{Re}\langle\hat{a}^2(T)\rangle$ is responsible for the occurrence of the RCP in $S_1(T)$, as discussed above. Now within the context of the above analysis the rescaled fluctuation factor for the cubic JCM, which can provide typical information on the $\langle\hat{\sigma}_z(T)\rangle_{m=1}$, is

$$Q_1(T) = \frac{S_1\left(\frac{2}{3}T\right) - \langle\hat{n}(0)\rangle}{\langle\hat{n}(0)\rangle}. \tag{31}$$

Figure 4 is given for $Q_1(T)$ that is represented by (31) for the same values of the parameters as those given in figure 2(a). Comparison between figures 2(b) and 4 is instructive. Actually, this is a novel result and its consequence is that the RCP of the $\langle\hat{\sigma}_z(T)\rangle_{m=1}$ can be obtained from the modified fluctuation factor of the cubic JCM for the same initial optical cavity field.

   Now we demonstrate the influence of atomic relative phases on the behaviour of $Q_1(T)$. Actually, in contrast to the natural phenomenon as well as the atomic inversion the rescaled fluctuation factor is insensitive to the values of the atomic relative phases. This fact can easily be recognized, where in the strong-intensity regime and for $\theta = \pi/4$, $\phi = 0$, one can show that $\mathrm{Re}\langle\hat{a}^2(T)\rangle$ includes such a term $[P(n)+P(n+m)]/2$, which cannot be zero for $P(n) \neq 0$. Therefore, (31) yields typical information on the atomic inversion provided that the atom is either in the excited state or in the ground state.

   From the above discussion generally the RCP occurring for the EHA cannot be established for the MEHA since for the latter $\mathrm{Re}\langle\hat{a}(T)\rangle \neq 0$ and $\mathrm{Im}\langle\hat{a}(T)\rangle \neq 0$ where the probability amplitudes of the wavefunction include non-trivial phase (cf (13)). Nevertheless, for particular type of states, e.g. parity states such as the even and odd coherent states, with strong initial mean-photon number the EHA and MEHA can provide almost similar behaviour. As in this case $\langle\hat{a}(T)\rangle = 0$, $\frac{\lambda h(n,m)}{\Omega_n} \simeq 1$ and also such type of terms, e.g., $\frac{(V_n-(n+m)\beta_2)}{\Omega_n} \simeq 0$. Therefore, the rescaled fluctuation factor for both, i.e. the EHA and MEHA, is almost similar except the $\mathrm{Re}\langle\hat{a}^2(T)\rangle$ in the MEHA involves $\cos(2T)$ additionally. However, for particular values of the initial mean-photon number, the maxima of $\cos(2T)$ occur in the course of the revival times of

$Q_1(T)$ and then the overall behaviour does not affect. The final remark: in the strong-intensity regime and when $\beta_1 = \beta_2 = \lambda = \omega$, the fluctuation factors of the MEHA—defined in the framework of the slowly varying operators–would be typically as those of the EHA defined in (15).

From the discussion given in this section, we can conclude that generally the EHA can be used to investigate the RCP for natural phenomenon approach but it is inadequate for numerical simulation approach. Nevertheless, for particular types of initial states—those for which the EHA and MEHA provide almost similar behaviour—the EHA is adequate also for numerical simulation approach.

4. The revival-collapse phenomenon in the amplitude-squared fluctuations

As we did in the previous section, we discuss briefly here whether the higher-order fluctuation factors can carry information on the corresponding atomic inversion or not. As an example we consider the amplitude-squared fluctuations [24]. The amplitude-squared fluctuations can occur in the fundamental mode in the second harmonic generation and can be converted into normal fluctuations. The two quadratures corresponding to the real and imaginary parts of the square of the field amplitude are

$$\hat{X}_2 = \frac{1}{4}[\hat{a}^2 + \hat{a}^{\dagger 2}], \qquad \hat{Y}_2 = \frac{1}{4i}[\hat{a}^2 - \hat{a}^{\dagger 2}]. \tag{32}$$

These quadratures obey the uncertainty relation

$$[\hat{X}_2, \hat{Y}_2] = \frac{1}{4i}(2\hat{a}^\dagger \hat{a} + 1). \tag{33}$$

After minor calculation, one can show that the two fluctuation factors associated with the amplitude-squared fluctuations are

$$\begin{aligned} F_2(T) &= \langle \hat{a}^{\dagger 2}(T)\hat{a}^2(T)\rangle + \text{Re}\langle \hat{a}^4(T)\rangle - 2(\text{Re}\langle \hat{a}^2(T)\rangle)^2, \\ S_2(T) &= \langle \hat{a}^{\dagger 2}(T)\hat{a}^2(T)\rangle - \text{Re}\langle \hat{a}^4(T)\rangle - 2(\text{Im}\langle \hat{a}^2(T)\rangle)^2, \end{aligned} \tag{34}$$

it is said that the system is able to yield amplitude-squared fluctuation when $F_2(T) < 0$ or $S_2(T) < 0$. Similar to section 3, we consider two approaches, which are natural phenomenon and numerical simulation. These will be discussed in the following. As the comparison between the EHA and MEHA leads to conclusions similar to those given in section 3 we will not discuss this issue in the present section.

4.1. Natural phenomenon

In this section, we are seeking states, which evolve with the standard JCM, say, in such a way that the contribution of the moments $\langle \hat{a}^2(T)\rangle$ and $\langle \hat{a}^4(T)\rangle$ to the fluctuation factors (34) is negligible in the course of the interaction. For such states expressions (34) reduce to

$$F_2(T) = S_2(T) = \langle \hat{a}^{\dagger 2}(T)\hat{a}^2(T)\rangle. \tag{35}$$

In fact, the quantity $\langle \hat{a}^{\dagger 2}(T)\hat{a}^2(T)\rangle$ can provide behaviour similar to that associated with the mean-photon number, i.e. atomic inversion. We have already introduced a class of states, which can fulfil the above requirements. That is the k-photon coherent states given by (5) for $k = 3, 5, 7, \ldots$ and the probability amplitudes are real. Here we give some details about the evolution of the third-photon coherent states with the standard JCM when the atom is initially in the excited atomic state. For this case one can easily show that

$$F_2(T) = \langle \hat{n}(0)\rangle^2 - \langle \hat{n}(0)\rangle \sum_{n=0}^{\infty} P(n)\cos(2T\sqrt{3n+4}), \tag{36}$$

where $\langle \hat{n}(0) \rangle$ is the initial mean-photon number of the third-photon coherent state. On the other hand, the corresponding atomic inversion is

$$\langle \hat{\sigma}_z(T) \rangle = \sum_{n=0}^{\infty} P(n) \cos(2T\sqrt{3n+1}). \tag{37}$$

In the strong-intensity regime expressions (36) and (37) yield

$$\langle \hat{\sigma}_z(T) \rangle \simeq \frac{\langle \hat{n}(0) \rangle^2 - F_2(T)}{\langle \hat{n}(0) \rangle}. \tag{38}$$

Argument similar to that given for (22) shows that the revival time of the present case can be obtained through the relation

$$2T_R[\sqrt{\langle \hat{n}(0) \rangle + 3} - \sqrt{\langle \hat{n}(0) \rangle}] \simeq 2\pi, \tag{39}$$

which leads to $T_R = \frac{2\pi}{3}\sqrt{\langle \hat{n}(0) \rangle}$, i.e. it is three times smaller than that associated with the initial coherent state case.

4.2. Numerical simulation

Similar arguments as those given in section 3 show that amplitude-squared fluctuation factors (34) of the mth ($m > 2$) JCM can exhibit behaviour similar to that of $\langle \hat{\sigma}_z(T) \rangle_{m=1}$ only when the values of $\text{Re}\langle \hat{a}^2(T) \rangle$ and $\text{Im}\langle \hat{a}^2(T) \rangle$ are very small (or zeros). In this case the forms of $\text{Re}\langle \hat{a}^4(T) \rangle$ and $\langle \hat{\sigma}_z(T) \rangle_{m=1}$ have to be comparable. Using similar procedures as those given in section 3 one can deduce the proportionality factor as

$$f(n) = mn^{\frac{m-3}{2}}. \tag{40}$$

Expression (40) indicates that $S_2(T)$ can provide behaviour similar to that of $\langle \hat{\sigma}_z(T) \rangle_{m=1}$ only when $m = 3$. This is similar to that associated with the normal fluctuation but here $f(n) = 3$. One can deduce the corresponding rescaled amplitude-squared fluctuation factor of the third JCM case, which includes behaviour typical to that of $\langle \hat{\sigma}_z(T) \rangle_{m=1}$ as

$$Q_2(T) = \frac{S_2\left(\frac{1}{3}\right) - \langle \hat{n}(0) \rangle^2}{\langle \hat{n}(0) \rangle^2}. \tag{41}$$

Comparison between (31) and (41) shows that the interaction time in (31) is two times greater than that in (41) owing to the fact that we deal with the square of the field amplitude. Finally, similar to the normal-fluctuation case the rescaled amplitude-squared fluctuation factor (41) is insensitive to the values of the relative phases of the atomic system.

5. Conclusions

In the present paper, we have discussed the possibility of relating the information involved in the fluctuation factors of the mth JCM to the atomic inversion of the EHA. We have made some comments on the differences between the EHA and MEHA. Generally, we have shown that there are two approaches, namely, natural phenomenon and numerical simulation. For the natural-phenomenon approach we have shown that there is a class of states for which fluctuation factors can include information on the corresponding atomic inversion naturally. This has been shown not only for normal fluctuations but also for amplitude-squared fluctuations. Furthermore, for such an approach fluctuation factors can exhibit coherent trapping based on the values of the relative phases of the atomic system. On the other hand, for the numerical-simulation approach we have shown that for specific value of m, in particular $m = 3$, the

fluctuation factors (or one of them) of the normal fluctuations as well as the amplitude-squared fluctuations can include the RCP similar to that associated with the atomic inversion of the standard JCM. More illustratively, the evolution of the quadrature fluctuations of an initially given field state interacting with a two-level system by a three-photon transition reflects the RCP phenomenon of the hypothetical interaction of the same field state with a two-level system by one-photon interaction where the level spacing is one-third of that of the former system. Furthermore, we have deduced the forms of the rescaled fluctuation factors for this case, which can involve typical information on the atomic inversion of the standard JCM. These forms would be helpful for experimentalists. In contrast to the natural approach, fluctuation factors here are insensitive to the values of the relative phases of the atomic system.

In fact these results are novel and indicate that the homodyne detector [25] can be used to measure the RCP. In this respect the signal coming from the microwave cavity is optically mixed with a strong coherent local oscillator using 50:50 beam splitter. Then the emerging fields are detected and the photocurrents are electronically treated in such a way that the measured quantity is the rescaled fluctuation factors. Quite recently, a similar set-up was given for measurement induced and quantum computation with atoms in optical cavities [26]. Moreover, in cavity QED, the homodyne detector technique has been applied for the single Rydberg atom and one-photon field aimed at studying the evolution of the field phase for the regular JCM [27]. Nevertheless, for the nonlinear version of the JCM in an ideal cavity ($Q = \infty$), e.g. the two-photon JCM, the detuning parameter $\triangle$ should be much greater than the Rabi frequencies of the one-photon transition ($\triangle = 33.3$ MHz in Cs, $\triangle = 39$ MHz in ^{85}Rb); thus the Stark shift and the two-photon coupling are appreciable [28]. Moreover, the progress in the trapped ions [29] and micromaser [30] is promising to produce the phenomenon presented in this paper. This is related to the fact that the two-photon Rydberg atom has already been realized in the micromaser [31]. We hope in the near future that it would be possible to produce a frequency within the range allowed by the equation $\omega = \omega_1 + \omega_2 + \omega_3$ where $\hbar\omega$ is the energy difference between the two levels and $\omega_1, \omega_2, \omega_3$ are the frequencies of the three photons generated by the transition.

Acknowledgments

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